

The $\Delta I = 1/2$ Rule for Kaons¹

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Abstract

We report on recent advances at understanding the $\Delta I = 1/2$ rule for kaons. We get reasonable matching between short- and long-distances for scales between 0.6 and 1.0 GeV and reproduce the $\Delta I = 1/2$ rule huge enhancement in the chiral limit. A detailed analysis of the different contributions to the relevant octet and 27-plet couplings is done. For the $B_6^{(1/2)}(\mu) \equiv \langle (\pi\pi)_0 | Q_6 | K \rangle / [\langle (\pi\pi)_0 | Q_6 | K \rangle]_{N_c}$ parameter, we get in the chiral limit $B_6^{(1/2)}(\mu) = 2.2 \pm 0.5$ for scales $\mu \in [0.6, 1.0]$ GeV.

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1 Introduction

Understanding the $\Delta I = 1/2$ rule for kaons within QCD has been a continuous challenge, see [1] for a review. Here, we report on recent work and advances at understanding this empirical rule in the chiral limit [2]. Due to the lack of space, we would like to concentrate on two main issues. The first one is the heavy X_i -bosons technique and the matching procedure. The second one is the anatomy of the different contributions to the octet coupling enhancement. We also give results on the penguin operator Q_6 which are relevant for ε'/ε .

Within the Standard Model (SM), $K \rightarrow \pi\pi$ decay amplitudes can be decomposed into definite isospin 0 and 2 amplitudes as follows [$A \equiv -iT$],

$$\begin{aligned} A[K_S \rightarrow \pi^0\pi^0] &\equiv \sqrt{\frac{2}{3}}A_0 - \frac{2}{\sqrt{3}}A_2, \\ A[K_S \rightarrow \pi^+\pi^-] &\equiv \sqrt{\frac{2}{3}}A_0 + \frac{1}{\sqrt{3}}A_2, \\ A[K^+ \rightarrow \pi^+\pi^0] &\equiv \frac{\sqrt{3}}{2}A_2. \end{aligned} \tag{1}$$

Where we have included the final state interaction phases δ_0 and δ_2 into the amplitudes A_0 and A_2 as follows

$$A_{0(2)} \equiv -ia_{0(2)}e^{i\delta_{0(2)}}. \tag{2}$$

Performing a fit to experimental data on $K \rightarrow \pi\pi$ and $K \rightarrow \pi\pi\pi$ up to Chiral Perturbation Theory (CHPT) order p^4 , in ref. [3] obtained

$$\left| \frac{A_0}{A_2} \right|^{(2)} = 16.4 \tag{3}$$

to lowest order. Unfortunately no fit uncertainties were quoted. This is the so-called $\Delta I = 1/2$ rule for kaons.

At $O(p^2)$ in CHPT, $|\Delta|S = 1$ amplitudes can be described in terms of three couplings in octet symmetry,

$$\begin{aligned} \mathcal{L}_{\Delta S=1}^{(2)} &= -\frac{3G_F}{5\sqrt{2}}V_{ud}V_{us}^*F_0^4 [G_8 \langle u_\mu u^\mu \Delta_{32} \rangle \\ &+ G'_8 \langle \chi_{(+)} \Delta_{32} \rangle \\ &+ G_{27} t^{ij,kl} \langle u^\mu \Delta_{ij} \rangle \langle u_\mu \Delta_{kl} \rangle] + \text{h.c.} \end{aligned} \tag{4}$$

We have pulled out the Fermi coupling constant, G_F , and the relevant Cabibbo-Kobayashi-Maskawa matrix elements V_{ij} . $U \equiv uu \equiv e^{i\sqrt{2}\Phi/F_0}$ with Φ a SU(3) matrix collecting the lowest pseudo-scalar meson π , K , and η_8 fields; F_0 is the

chiral limit value of the pion decay constant $f_\pi \simeq 92.4$ MeV; $D_\mu U$ is the covariant derivative acting on U and $u_\mu \equiv iu^\dagger(D_\mu U)u$; $\chi_{(+)} \equiv u^\dagger \chi u^\dagger + u \chi^\dagger u$ with $\chi \equiv 2B_0 \mathcal{M}$, \mathcal{M} is a 3×3 matrix collecting the light quark masses and B_0 is proportional to the quark condensate in the chiral limit, $B_0 \equiv -\langle 0 | \bar{q}q | 0 \rangle / F_0^2$. The symbols Δ_{ij} and 27-plet tensor $t^{ij,kl}$ take into account for the correct flavour combinations and were defined in [4]. At this order

$$\left| \frac{A_0}{A_2} \right|^{(2)} = \sqrt{2} \left(\frac{9G_8 + G_{27}}{10G_{27}} \right). \quad (5)$$

At leading order in $1/N_c$, $G_8 = G_{27} = 1$ and

$$\left| \frac{A_0}{A_2} \right|^{(2)} = \sqrt{2}; \quad (6)$$

i.e. more than a factor ten lower than the experimental number !

2 The Heavy X_i -Bosons Method: Matching Short- and Long-Distances

We analyse $|\Delta S| = 1$ off-shell two-point Green functions

$$\begin{aligned} & \Pi^{ij}(q^2) \\ & \equiv i \int d^4x e^{iqx} \langle 0 | T \{ P^i(0)^\dagger P^j(x) e^{i\Gamma_{\Delta S=1}} \} | 0 \rangle \end{aligned} \quad (7)$$

in the presence of strong interactions. These Green functions were studied in CHPT to $O(p^2)$ in [5] and to $O(p^4)$ in [4]. $P^i(x)$ are external pseudo-scalar sources that couple to pion, kaon, and η_8 fields. The $\Delta S = 1$ Standard Model effective action at some scale μ below the charm quark mass, can be written as

$$\Gamma_{\Delta S=1} \equiv -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_{i=1} C_i(\mu) \int d^4y Q_i(y) \quad (8)$$

with $C_i(\mu)$ Wilson coefficients which are known to two-loops and $Q_i(y)$ are four-quark local operators inducing $\Delta S = 1$ transitions. The list of relevant operators is given in [2].

The effective action $\Gamma_{\Delta S=1}$ is generated by *virtual* W -boson exchanges. This makes necessary the intervention of strong interactions at *all* scales between 0 and ∞ to calculate weak matrix elements. Matching long- and short-distances is the big challenge of calculating weak matrix elements. The procedure we propose is to use an effective field theory which reproduces the physics of the four-quark

$\Gamma_{\Delta S=1}$ operators *below* some scale μ_L around the charm quark mass through the exchange of heavy X_i -bosons. For instance,

$$X_\mu^1 \{g_1(\mu_L, M_X, \dots)[\bar{s}_L \gamma^\mu d_L] + g'_1(\mu_L, M_X, \dots)[\bar{u}_L \gamma^\mu u_L]\} \quad (9)$$

reproduces the physics of $Q_1(x)$ *below* μ_L . At μ_L we need matching conditions which fix $g_1(\mu_L, M_X, \dots)$ and $g'_1(\mu_L, M_X, \dots)$. A detailed example on how this procedure works will be presented in [6].

In this way, we resum large $\log(M_W/\mu)/N_c$ to any available order in the short-distance and long-distance calculations. Scale and scheme dependence are correctly treated as well.

In the heavy X_i -boson exchange effective field theory, the basic non-leptonic interaction for physics below μ_L is given by

$$\sim g_1^\dagger g'_1 \int \frac{d^4 r}{(2\pi)^4} \int e^{iq \cdot x} \frac{g_{\mu\nu}}{M_{X_1}^2} J_1^{\mu\dagger}(x) J_1^{\nu'}(0). \quad (10)$$

Now, we can calculate analogously to what one does for the γ -exchange contribution to $\pi^+-\pi^0$ or K^+-K^0 [7] mass difference. We can separate the long- and short-distance pieces using an Euclidean cut-off μ

$$\int d^4 r_E \rightarrow \int d\Omega \left[\int_0^\mu d|r_E| + \int_\mu^\infty d|r_E| \right]. \quad (11)$$

The short-distance piece is then consistently treated within the heavy X_i -boson exchange effective theory at next-to-leading order in the $1/N_c$ expansion and the long-distance piece with an appropriate hadronic model or data if available. In our case, the low-energy part is treated with the ENJL model presented in [8]. At leading order in $1/N_c$, this is a model with the same chiral structure as QCD and which reproduces most of the low energy dynamics of the strong interactions. These two features together with a reasonable matching with short-distance QCD are expected to be the bulk of the dynamics needed to predict weak matrix elements and are the basis of our calculational method while lacking first principle calculations. Model dependence enters only through the evaluation of the long-distance piece in (11).

The ENJL model doesn't confine and does have a wrong high energy behaviour at high energies. We smear out these bad features by calculating far off-shell with very small momenta and using only fits up to order five or six at most, see more details in [2]. There are good prospects to eliminate to a large extent the bad high energy behaviour and enlarge beyond 1 GeV the matching between short- and long-distances using the model in [9].

3 The $\Delta I = 1/2$ Rule

Here we give the main conclusions of our work. Penguin-like diagrams with Q_2 dominate the octet coupling G_8 (around 63 %) in the whole range of scales studied [between 0.5 GeV and 1. GeV] producing the observed huge enhancement. The penguin-operator Q_6 contribution to G_8 is around 12 %.

There is a large cancellation between the B_K -like diagrams contribution to G_8 from Q_1 and Q_2 . The relatively large positive contribution from Q_1 is canceled by B_K -like diagrams from Q_2 to give in total less than 7 % of G_8 from B_K -like diagrams. Factorizable contributions plus B_K -like contributions are around 23 % of G_8 . The sum of the rest of operators contributes by less than 5 % and decreases G_8 up to its final value. More than 75 % of the value of G_8 comes from penguin-like diagrams. We show in Figure 1 the matching obtained for the three $O(p^2)$ couplings and in Figure 2 the relative contributions of Q_1 , Q_2 , and Q_6 to G_8 .

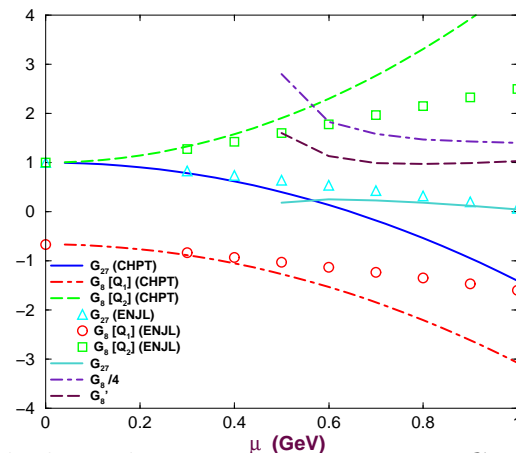


Figure 1: We show the long-distance contributions to G_8 , G_{27} , and G'_8 couplings using lowest order CHPT (quadratic dependence in μ) and ENJL. The final scheme independent result including short-distance at two-loops and matching is also shown.

At the same time, there are no penguin-like contributions to G_{27} and B_K -like diagrams for Q_1 and Q_2 decrease the 27-plet coupling from one to a value between one half and one third. There is a large cancellation between the contributions of Q_1 and Q_2 . In summary, penguin-like diagrams with Q_2 dominate largely the enhancement of G_8 and B_K -like diagrams for $Q_1 + Q_2$ produce the small value of G_{27} . These two facts are responsible for the $\Delta I = 1/2$ rule in (3).

Experimentally

$$G_8 = 6.2 \pm 0.7; \quad G_{27} = 0.48 \pm 0.06. \quad (12)$$

Here, we have only included the uncertainty from the value of the pion decay constant in the chiral limit $F_0 = (86 \pm 10)$ MeV, since no uncertainties from the

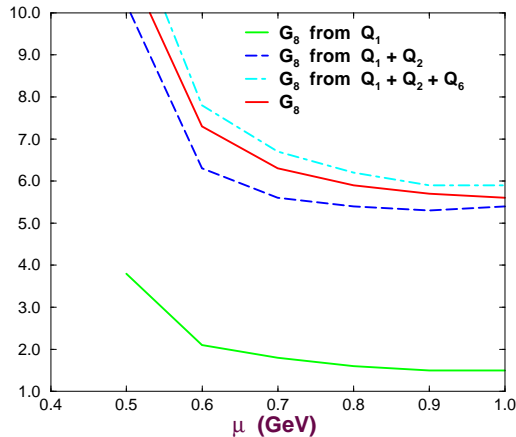


Figure 2: The contributions of Q_1 , $Q_1 + Q_2$, $Q_1 + Q_2 + Q_6$ to G_8 . The final result for G_8 using short-distance at two-loops with the scheme dependence removed [2, 6] is also shown.

fit procedure were quoted in [3]. We get

$$\begin{aligned} 4.3 &< G_8 < 7.5; & 0.8 < G'_8 < 1.1; \\ 0.25 &< G_{27} < 0.40 \end{aligned} \quad (13)$$

and

$$15 < \left| \frac{A_0}{A_2} \right|^{(2)} < 40. \quad (14)$$

This last result is somewhat large mainly because of the small value we get for G_{27} . Notice that this calculation is to next-to-leading in $1/N_c$ and to *all* orders in Chiral Perturbation Theory. One can expect, therefore non-negligible $1/N_c^2$ corrections –typically of the order of (30~40) %, but the main $\Delta I = 1/2$ enhancement is there. We want to stress that these are parameter free *predictions*, the three input values we need were fixed in [8] from low energy phenomenology in the strong sector. We believe there are good prospects at obtaining *predictions* on $\Delta S = 1$ transitions and ε'/ε [10].

4 The Q_6 Penguin Operator

In the chiral limit, the contribution of Q_6 to G_8 is proportional to the quark condensate squared. At leading order in $1/N_c$, the scale dependence of $\langle \pi\pi|Q_6|K \rangle$ is exactly canceled by the Wilson coefficient $C_6(\mu)$ [12]. We have shown in [2] that the scale dependence is also canceled at next-to-leading in $1/N_c$ for the factorizable part. Then, as for the rest of $Q_i(y)$ operators, the matching between short- and long-distances becomes an affair of non-factorizable contributions.

Outside the chiral limit, the strange quark condensate squared *does not* factorize and most of its quark mass corrections produce actually the coupling G'_8 which does not contribute to $K \rightarrow \pi\pi$. Both G_8 and G'_8 are still proportional to the chiral limit value of the quark condensate squared, and kaon and pion masses enter in higher order CHPT corrections. Therefore, the contribution to G_8 from Q_6 cannot be proportional to $1/m_s^2$ and the usual parameterization being inversely proportional to the strange quark mass squared is very misleading. In addition, the VSA result of $\langle(\pi\pi)_0|Q_6|K\rangle$ has an IR divergence as shown in [2]. In view of all these problems, we propose to quote directly values of matrix elements as we did in [2]. We give in Table 1 the results for the contribution of Q_6 to G_8 .

Scale (GeV)	One-Loop	Two-loops (SI)
0.5	0.98	2.14
0.6	0.73	1.49
0.7	0.53	1.10
0.8	0.38	0.83
0.9	0.26	0.62
1.0	0.15	0.45

Table 1: The contribution of Q_6 to G_8 using short-distance to one-loop and to two-loops with the the scheme dependence removed see [2, 6].

However, for the sake of comparison with other results in the literature which only quote B_6 -parameters, we give our results for

$$B_6^{(1/2)}(\mu) \equiv \langle(\pi\pi)_0|Q_6|K\rangle / [\langle(\pi\pi)_0|Q_6|K\rangle|_{N_c}] \quad (15)$$

where

$$\begin{aligned} \langle(\pi\pi)_0|Q_6|K\rangle|_{N_c} = & -i 32 \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* C_6(\mu) \\ & \times F_0(m_K^2 - m_\pi^2) \frac{\langle 0|\bar{q}q|0\rangle^2(\mu)}{F_0^6} L_5(\nu). \end{aligned} \quad (16)$$

$\langle 0|\bar{q}q|0\rangle$ is the quark condensate in the chiral limit which in a very good approximation we take to be the average of up and down quark condensate [13]. With this definition, we avoid the IR divergence in the VSA value of $\langle(\pi\pi)_0|Q_6|K\rangle$ [2]. The large N_c result (16) contains still the ambiguity in the value of the scale ν which is only canceled by the IR divergent part. We fix $\nu = M_\rho$ and use $L_5(M_\rho) = (1.4 \pm 0.3) 10^{-3}$.

To lowest CHPT order p^2 and next-to-leading in $1/N_c$, we get

$$B_6^{(1/2)}(\mu) = 0.76 \pm 0.20 \quad (17)$$

in agreement with [11]. To this order the scale dependence μ is canceled exactly due to the need of canceling the IR divergence. Notice also that the value of $B_6^{(1/2)}$ very near to one is due to the large cancellation between the two types of factorizable contributions [2]. This cancellation is exact at order p^0 and very large at order p^2 due to the cancellation of the IR divergence. It does not however protect the value of $B_6^{(1/2)}$ from higher CHPT order corrections. In fact, also in the chiral limit but to *all* orders in CHPT and next-to-leading in $1/N_c$, we get

$$B_6^{(1/2)}(\mu) = 2.2 \pm 0.5 \quad (18)$$

for scales $\mu \in [0.6, 1.0]$ GeV. The scale dependence is very mild. The importance of higher order CHPT corrections is manifest in this result. A large enhancement of $B_6^{(1/2)}$ was also obtained in [11] when $O(p^4)$ corrections were included.

We found that the G_8 enhancement is not due to the contribution of the penguin operator Q_6 . The only relation between the dynamics underlying the value of ε'/ε in the SM and the large value of G_8 is the type of dominant diagrams, namely, penguin-like diagrams. But, ε'/ε is dominated by the penguin operators Q_6 and Q_8 while G_8 by the Q_2 operator.

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